INDIAN MARITIME UNIVERSITY

(A Central University, Government of India)

End Semester Examination Dec 2019/Jan 2020

B.Tech (Marine Engineering)

Semester -III

UG11T3301- Computational Mathematics

Date: 10.12.2019	**Max Marks: 70**
Time: 3 Hours	**Pass Marks: 35**

Part– A **(10 x2 = 20 marks)**

Compulsory Questions: (The symbols have their usual meanings.)

1. If a population is normally distributed with mean 100 and standard deviation 16, then what is the mean and standard deviation of the sampling distribution of sample mean $\bar{X}$ for random samples of size 4.
2. Derive the normal equations required to fit the curve $xy^a = b$ in given set of values of x and y.
3. Two lines of regression are given by $8x - 10y + 66 = 0$ and $40x - 18y = 214$. If variance x is 9, find
   (i) Correlation coefficient between x and y (ii) the standard deviation of y.
4. Prove the axiom $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ of Boolean algebra by means of truth table.
5. Find the missing term in the following table:

x	0	1	2	3	4
y	1	3	9	$-$	81

6. Using the shift operator E, derive the Newton's forward interpolation formula for the function $f(x_0 + ph)$ where $p = (x - x_0)/h$.
7. Evaluate the integral $\int_0^6 \frac{1}{1+x^2} dx$ using the Simpson's (3/8) rule with 6 equal sub-intervals.
8. Solve the difference equation of the form $y_{n+2} - 5y_{n+1} - 6y_n = 0$
9. Find an approximate value of root of the equation $x^3 + x - 1 = 0$ near $x = 1$ by using Regula Falsi method up to two iterations.
10. Explain bubble short method with suitable example.

Part– B **(5 x 10 = 50 marks)**

Answer any FIVE of the following seven questions

11. (a) The voltage v across a capacitor at time t seconds is given by the following table:

t	0	2	4	6	8
v	150	63	28	12	5.6

Use the method of least square to fit a curve of the form $v = ae^{kt}$ to this data. [5 marks]

(b) Find a parabola of the form $y = a + bx + cx^2$ which fits most closely to the following observations: [5 marks]

x	-3	-2	-1	0	1	2	3
y	4.63	2.11	0.67	0.09	0.63	2.15	4.58

12. (a) Express the value of θ in terms of x using following data

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

Also find θ at $x = 43$. [4+1=5 marks]

(b) Express the function $\frac{x^2+6x-1}{(x-1)(x+1)(x-4)(x-6)}$ as sum of partial fractions by using Lagrange's interpolation formula. [5 marks]

13. (a) The table below reveals the velocity v of a body during the specific time t, find its acceleration at $t = 1.1$. [5 marks]

t	1.0	1.1	1.2	1.3	1.4
v	43.1	47.7	52.1	56.4	60.8

(b) The velocity v (km/min) of a moped which starts from rests, is given at fixed intervals of time t (min) as follows:

t	2	4	6	8	10	12	14	16	18	20
v	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes. [5 marks]

14. (a) If x and y are two random variables with same standard deviation and correlation coefficient r. Show that the correlation coefficient of x and $x + y$ is $\sqrt{\frac{1+r}{2}}$. [5 marks]

(b) Prove that $u_0 + u_1 + u_2 + \ldots n-$ terms $= nu_0 + \frac{n(n-1)}{2!}\Delta u_0 + \frac{n(n-1)(n-3)}{3!}\Delta^2 u_0 + \ldots \infty$. Hence sum the series $2 \cdot 5 + 5 \cdot 8 + 8 \cdot 11 + 11 \cdot 14 + \cdots n-\text{terms}$. [5 marks]

15. (a) Solve the difference equation $u_{n+2} - 7u_{n+1} + 10u_n = 12e^{3n} + 4^n$. [5 marks]

(b) Write an algorithm to find factorial of a positive integer and draw its flow chart. [5 marks]

16. (a) Solve by Taylor's series method the differential equation $\frac{dy}{dx} = \log xy$ for $y(1.1)$ and $y(1.2)$, given $y(1) = 2$. [5 marks]

(b) Using Runge-Kutta method of fourth order, solve the differential equation $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with $y(0) = 1$ at $x = 0.2,\ 0.4$. [5 marks]

17. (a) In a Boolean algebra, prove that:

(i) $(x \wedge y') \vee (x' \wedge y) \vee (x' \wedge y') = x' \vee y'$,

(ii) $(x \vee y) \wedge (x \vee y') \wedge \left(x' \vee y\right) = x \wedge y$. [3+3=6 marks]

(b) Draw the circuit diagrams for the Boolean functions

(i) $f = p_1 \wedge [(p_2 \wedge p_3) \vee [p_4 \wedge (p_5 \vee p_6)]]$

(ii) $f = (p_1 \vee p_2) \wedge [(p_3 \wedge p_4) \vee [(p_5 \vee p_6) \wedge p_7]]$ [2+2=4 marks]

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